

The fixpoint has been reached, thus this relation contains all state pairs that satisfy both SIM1 and SIM2.

Also, SIM3 holds, i.e., for each initial state of the first structure, there is an initial state in the second one that simulates it. Hence, the second structure simulates the first one.

SIM3 does not hold for the following initial states of the first Kripke structure $\{Q_2\}$.

$\neg AXG\neg a = EXGFa$ $AFXGa = AXFGa$	a b e f
$AG(\overline{X}a \leftrightarrow a) = AGa$ $AG(a \leftrightarrow Xa) = A(Ga \vee G\neg a)$	c d g h
$F[Fa \cup b] = F[Fa \cup b] \vee GFa = Fb \vee GFa$ $F[a \cup b] = F[a \cup b] \vee FGa = Fb \vee FGa$	e=1/a f=b/-a
$E[a \cup b \cup (Ec)] = E[a \cup b \cup c]$ $E[a \cup b \cup (Ec)] = E[a \cup b \cup c]$	g=c/a h=d+cf
$A[a \cup B \cup (Bc)] = A[a \cup B \cup c]$ $= A[(a \cup B \cup c) \vee G\neg(b \cup c)]$	

\mathcal{L}_1 can be encoded as $GF(\overline{X}(\neg a \wedge \overline{X}(a \wedge \overline{X}\neg a)))$, which is equivalent to $GF(\neg a \wedge X(a \wedge X\neg a))$.

$$\begin{aligned}
 & GF(\overline{X}(\neg a \wedge \overline{X}(a \wedge \overline{X}\neg a))) \\
 &= \mathcal{A}_\exists(\{q_1\}, \\
 &\quad q_1, \\
 &\quad (Xq_1 \leftrightarrow \neg a), \\
 &\quad GF(\overline{X}(\neg a \wedge \overline{X}(a \wedge q_1)))) \\
 &= \mathcal{A}_\exists(\{q_1, q_2\}, \\
 &\quad q_1 \wedge q_2, \\
 &\quad (Xq_1 \leftrightarrow \neg a) \wedge (Xq_2 \leftrightarrow (a \wedge q_1)), \\
 &\quad GF(\overline{X}(\neg a \wedge q_2))) \\
 &= \mathcal{A}_\exists(\{q_1, q_2, q_3\}, \\
 &\quad q_1 \wedge q_2 \wedge q_3, \\
 &\quad (Xq_1 \leftrightarrow \neg a) \wedge (Xq_2 \leftrightarrow (a \wedge q_1)) \wedge (Xq_3 \leftrightarrow (\neg a \wedge q_2)), \\
 &\quad GFq_3)
 \end{aligned}$$

Check whether $\mathcal{K} \models A\varphi$ holds by means of LTL model checking:

$$\begin{aligned}
 \mathcal{K} \models A\varphi & \\
 \Leftrightarrow \mathcal{K} \models \neg E\neg\varphi & \\
 \Leftrightarrow \mathcal{K} \models \neg E\mathcal{L}\neg\varphi & \\
 \Leftrightarrow \mathcal{K}_x \models \neg E\varphi_{\mathcal{F}} & \\
 \Leftrightarrow \mathcal{K}_x \models \neg EGF(p \vee \neg a \wedge b) & \\
 [\neg EGF(p \vee \neg a \wedge b)]_{\mathcal{K}_x} = S \setminus \{ \}, & \text{ therefore we conclude that } \mathcal{K} \models A\varphi.
 \end{aligned}$$

Check whether $\mathcal{K} \models \neg A\varphi$ holds by model checking of \mathcal{L}_w :

$$\begin{aligned}
 \mathcal{K} \models \neg A\varphi & \\
 \Leftrightarrow \mathcal{K} \models E(\neg\varphi) & \\
 \Leftrightarrow \mathcal{K} \models E\mathcal{L}\neg\varphi & \\
 \Leftrightarrow \mathcal{K} \models E\mathcal{A}_\exists(Q, \varphi_{\mathcal{I}}, \varphi_{\mathcal{R}}, \varphi_{\mathcal{F}}) & \\
 \Leftrightarrow \mathcal{K}_x \models E\varphi_{\mathcal{F}} & \\
 \Leftrightarrow \mathcal{K}_x \models EGF(p \vee \neg a \wedge b) & \\
 [EGF(p \vee \neg a \wedge b)]_{\mathcal{K}_x} = \{ \}, & \text{ therefore we conclude that } \mathcal{K} \not\models \neg A\varphi.
 \end{aligned}$$

$$\begin{aligned}
 G[a \cup b] \vee [c \cup B(\overline{F}b)] & \\
 = \mathcal{A}_\exists(\{q_0\}, & \\
 \neg q_0, & \\
 (Xq_0 \leftrightarrow b \vee a \wedge q_0), & \\
 G(b \vee a \wedge q_0) \vee [c \cup B(\overline{F}b)] & \\
 = \mathcal{A}_\exists(\{q_0, q_1\}, & \\
 \neg q_0, & \\
 (Xq_0 \leftrightarrow b \vee a \wedge q_0) \wedge (q_1 \leftrightarrow (b \vee a \wedge q_0) \wedge Xq_1), & \\
 (q_1 \vee [c \cup B(\overline{F}b)]) \wedge GF[(b \vee a \wedge q_0) \rightarrow q_1] & // q_1 \vee [c \cup B(\overline{F}b)] \\
 = \mathcal{A}_\exists(\{q_0, q_1, q_2\}, & \\
 \neg q_0 \wedge \neg q_2, & \\
 (Xq_0 \leftrightarrow b \vee a \wedge q_0) \wedge (q_1 \leftrightarrow (b \vee a \wedge q_0) \wedge Xq_1) & \\
 \wedge (Xq_2 \leftrightarrow b \vee q_2) \wedge (q_3 \leftrightarrow \neg(b \vee q_2) \wedge (c \vee Xq_3)), & \\
 (q_1 \vee q_3) \wedge GF[(b \vee a \wedge q_0) \rightarrow q_1] \wedge GF[q_3 \vee (b \vee q_2)] & // q_1 \vee q_3 \\
 = \mathcal{A}_\exists(\{q_0, q_1, q_2, q_3\}, & \\
 \neg q_0 \wedge \neg q_2, & \\
 (Xq_0 \leftrightarrow b \vee a \wedge q_0) \wedge (q_1 \leftrightarrow (b \vee a \wedge q_0) \wedge Xq_1) & \\
 \wedge (Xq_2 \leftrightarrow b \vee q_2) \wedge (q_3 \leftrightarrow \neg(b \vee q_2) \wedge (c \vee Xq_3)), & \\
 (q_1 \vee q_3) \wedge GF[(b \vee a \wedge q_0) \rightarrow q_1] \wedge GF[q_3 \vee (b \vee q_2)] & // 1 \\
 = \mathcal{A}_\exists(\{q_0, q_1, q_2, q_3\}, & \\
 \neg q_0 \wedge \neg q_2 \wedge (q_1 \vee q_3), & \\
 (Xq_0 \leftrightarrow b \vee a \wedge q_0) \wedge (q_1 \leftrightarrow (b \vee a \wedge q_0) \wedge Xq_1) & \\
 \wedge (Xq_2 \leftrightarrow b \vee q_2) \wedge (q_3 \leftrightarrow \neg(b \vee q_2) \wedge (c \vee Xq_3)), & \\
 GF[(b \vee a \wedge q_0) \rightarrow q_1] \wedge GF[q_3 \vee (b \vee q_2)] & // 1
 \end{aligned}$$

$$\begin{aligned}
 \Phi(X\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow X\varphi, \Phi(q)_x) \\
 \Phi(\overline{X}\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q_0, q_1\}, 1, (q_0 \leftrightarrow \varphi) \wedge (q_1 \leftrightarrow Xq_0), \Phi(q_1)_x) \\
 \Phi(G\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \varphi \wedge Xq, \Phi(q)_x \wedge GF[q \rightarrow \varphi]) \\
 \Phi(F\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \varphi \vee Xq, \Phi(q)_x \wedge GF[q \rightarrow \varphi]) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \psi \vee \varphi \wedge Xq, \Phi(q)_x \wedge GF[q \rightarrow \psi]) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \psi \vee \varphi \wedge Xq, \Phi(q)_x \wedge GF[q \rightarrow \psi]) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \neg\psi \wedge (\varphi \vee Xq), \Phi(q)_x \wedge GF[q \rightarrow \psi]) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, 1, q \leftrightarrow \neg\psi \wedge (\varphi \vee Xq), \Phi(q)_x \wedge GF[q \rightarrow \psi]) \\
 \Phi(\overline{X}\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \varphi, \Phi(q)_x) \\
 \Phi(\overline{X}\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \varphi, \Phi(q)_x) \\
 \Phi(\overline{X}\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \varphi \wedge q, \Phi(\varphi \wedge q)_x) \\
 \Phi(\overline{X}\varphi)_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \varphi \vee q, \Phi(\varphi \vee q)_x) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \psi \vee \varphi \wedge q, \Phi(\psi \vee \varphi \wedge q)_x) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \psi \vee \varphi \wedge q, \Phi(\psi \vee \varphi \wedge q)_x) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, q, Xq \leftrightarrow \neg\psi \wedge (\varphi \vee q), \Phi(\neg\psi \wedge (\varphi \vee q))_x) \\
 \Phi([\varphi \cup \psi])_x & \Leftrightarrow \mathcal{A}_\exists(\{q\}, \neg q, Xq \leftrightarrow \neg\psi \wedge (\varphi \vee q), \Phi(\neg\psi \wedge (\varphi \vee q))_x)
 \end{aligned}$$

ϕ_1 is inductive with respect to the given Kripke structure iff

Induction base : $\varphi_X \rightarrow \psi_1$ holds,
 Induction step : $\psi_1 \rightarrow \Box\psi_1$ holds with respect to $\varphi_{\mathcal{R}}$,
 i.e., ψ_1 holds in all the successors of the states that satisfy ψ_1 .

– Induction base can be proved.
 – To prove the induction step, we have to compute $\text{suc}_S^{\exists}(\psi_1)$, i.e., the existential successors of the states satisfying ψ_1 .

$$\begin{aligned}
 \text{suc}_S^{\exists}(\psi_1) & \\
 = [\exists a, b, c. (\neg a \wedge \neg b \wedge a' \wedge \neg c' \vee \neg a \wedge b \wedge \neg a' \wedge \neg c' \vee \neg a \wedge \neg b \wedge c \wedge a' \wedge \neg b' \wedge c') & \\
 \wedge (\neg a \vee b \vee \neg c)]_{a',b',c'}^{a,b,c} & \\
 = [(a' \wedge \neg c' \vee \neg a' \wedge \neg c' \vee a' \wedge \neg b' \wedge c')]_{a',b',c'}^{a,b,c} & \\
 = \neg c \vee a \wedge \neg b \wedge c & \\
 \text{suc}_S^{\exists}(\psi_1) \rightarrow \psi_1 & \text{ is not valid.}
 \end{aligned}$$

Therefore, we conclude that ψ_1 is not inductive with respect to the given Kripke structure.

$$\begin{aligned}
 & A(X[b \cup a] \vee F[a \cup Xb]) \\
 &= A(X[b \cup a] \vee XFb) \quad - F[a \cup Xb] \leftrightarrow FXb \leftrightarrow XFb \\
 &= AXFb \\
 &= AXAFb \quad - X[b \cup a] \vee XFb \leftrightarrow XFb \\
 &= \Box(\nu y. \Diamond y \rightarrow (\mu x. (b \vee \Box x)))
 \end{aligned}$$

$$\begin{aligned}
 & E([\neg a] W b] \wedge F[a \cup b]) \\
 &= E([\neg a] W b] \wedge Fb) \quad // \text{ Since } F[a \cup b] \leftrightarrow Fb. \\
 &= E([\neg a] W b] \quad // \text{ Since } ([\neg a] W b] \wedge Fb) \leftrightarrow ([\neg a] W b] \\
 & AF([1 \cup \neg a \vee \neg b]) \vee FGb) \\
 &= AF(\neg G(a \wedge b) \vee FGb) \in \mathcal{S}_2 \\
 &= AF(G(a \wedge b) \rightarrow FGb) \in \mathcal{S}_2 \\
 &= AF1 \quad // \text{ Since } G(a \wedge b) \text{ implies } FGb. \\
 & \neg AX(XG(\neg E[1 \cup (E\neg a)])) \\
 &= EXX(FG(E[1 \cup (E\neg a)])) \quad \varphi_3 \in \mathcal{S}_3 \\
 &= EXX(FG(E[1 \cup (\neg a)])) \\
 &= EFG(E[1 \cup (\neg a)]) \quad // \text{ Since } XXFGa \text{ is equal to } FGa. \\
 &= EFEG(E[1 \cup (\neg a)]) \\
 & AG\neg[a \cup Gb] \\
 &= AG[\neg a \cup Gb] \quad // \text{ Since } \neg[a \cup Gb] \text{ is equal to } [\neg a \cup Gb] \\
 &= AGA[\neg a \cup Gb] \\
 &= AGA[\neg a \cup B(EGb)] \quad \varphi_4 \in \mathcal{S}_2 \\
 & EFGa \rightarrow AFG\neg a \quad \varphi_5 \in \mathcal{S}_2 \\
 &= AGF\neg a \vee AFG\neg a \\
 &= AGF\neg a \quad // \text{ Since } AFGa \text{ implies } AGFa. \\
 &= AGAF\neg a
 \end{aligned}$$

CTL* has no restrictions:

state formulas: $S ::= V_\Sigma | \neg S | S \wedge S | EP | AP$
 path formulas: $P ::= S | \neg P | P \wedge P$
 $| XP | [P \cup P] | \overline{X}P | [P \overline{U} P]$

LTL has essentially only pure path formulas

state formulas: $S ::= AP$
 path formulas: $P ::= V_\Sigma | \neg P | P \wedge P$
 $| XP | [P \cup P] | \overline{X}P | [P \overline{U} P]$

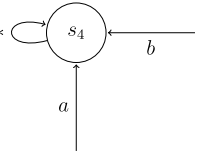
CTL has a restricted set of state formulas

state formulas: $S ::= V_\Sigma | \neg S | S \wedge S$
 $| EXS | E[S \cup S] | AXS | A[S \cup S]$

$$\begin{aligned}
 & [(Gb) \overline{U} c] \wedge GXFa \\
 &= \mathcal{A}_\exists(\{q_0\}, \\
 &\quad 1, \\
 &\quad (q_0 \leftrightarrow b \wedge Xq_0), \\
 &\quad [q_0 \overline{U} c] \wedge GXFa) \\
 & // [q_0 \overline{U} c] \wedge GXFa \wedge GF(b \rightarrow q_0) \\
 &= \mathcal{A}_\exists(\{q_0, q_1\}, \\
 &\quad q_1, \\
 &\quad (q_0 \leftrightarrow b \wedge Xq_0) \wedge (Xq_1 \leftrightarrow c \vee q_0 \wedge q_1), \\
 &\quad (c \vee q_0 \wedge q_1) \wedge GXFa) \\
 & // (c \vee q_0 \wedge q_1) \wedge GXFa \wedge GF(b \rightarrow q_0) \\
 &= \mathcal{A}_\exists(\{q_0, q_1, q_2\}, \\
 &\quad q_1, \\
 &\quad (q_0 \leftrightarrow b \wedge Xq_0) \wedge (Xq_1 \leftrightarrow c \vee q_0 \wedge q_1) \wedge (q_2 \leftrightarrow a \vee Xq_2), \\
 &\quad (c \vee q_0 \wedge q_1) \wedge GXq_2) \wedge GF(q_2 \rightarrow a) \\
 & // (c \vee q_0 \wedge q_1) \wedge GXq_2 \wedge GF(b \rightarrow q_0) \wedge GF(q_2 \rightarrow a) \\
 & - - -
 \end{aligned}$$

To apply the Rabin/Scott construction, we first have to make the automaton totally defined, i.e., every input should be readable in every state. To this end, we add a sink state s_4 :

With this sink state, we use the acceptance condition $F(s_1 \vee s_3) \wedge G\neg s_4$ to have an equivalent prefix automaton. **The acceptance condition is no longer just an F-condition. In addition, we have to state that we never reach the states that have only the sink state.**



The given automaton is totally defined, and the acceptance condition $\varphi_{\mathcal{F}} = Fq_2$ defines the accepting states $\mathcal{F} = \{s_2, s_3\}$. We can apply the subset construction. The new acceptance condition is $F(Q_2 \vee Q_3 \vee Q_4 \vee Q_5)$.

